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Abstract

In this paper we modify a standard quality ladder model by assuming that R&D is driven by outsider firms and the winners of the race sell licenses over their patents, instead of entering directly the intermediate good sector. As a reward they get the aggregate profit of the industry. Moreover, in the intermediate good sector firms compete à la Cournot and it is assumed that there are spillovers represented by strategic complementarities on costs. We prove that there exists an interval of values of the spillover parameter such that the relationship between competition and growth is an inverted-U-shape.

Keywords: quality ladder; Cournot oligopoly; strategic complementarities; competition.

JEL Classification: L13, L16, O31, O52.

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1 Introduction

Empirical analysis has recently provided evidence in favor of an inverted U-shaped relationship between competition and growth (see Aghion *et al.* 2005); nonetheless, only few theoretical models of growth and innovation are capable of explaining such empirical evidence. This paper proposes a novel rationale for the inverted U-shaped relationship, stemming from a modified quality ladder model in which we assume that firms compete à la Cournot in the intermediate good sector, where positive externalities or spillovers on costs are present. It is just the presence of a spillover effect which justifies the fact that a higher product market competition may enhance growth, because it can influence positively the profits that reward innovators.

Standard Industrial Organization theory (Salop 1977, Dixit and Stiglitz 1977) and the first generation of Schumpeterian growth models (Grossman and Helpman 1991; Aghion and Howitt 1992; Barro and Sala-i-Martin 2004) predict that innovation and hence growth should decline with competition, because more competition reduces the rents that reward successful innovators. This discourages firms from investing in R&D, thus reducing the innovation rate and as a consequence the long run growth rate of the economy. However, the empirical literature, as Gerosky (1995), Nickell (1996) and Blundell *et al.* (1999), suggests a positive correlation between competition and growth. The theoretical literature tried to solve this dilemma by modifying radically the assumptions of the basic Neoschumpeterian model.

For example Aghion *et al.* (1999) introduce agency considerations in the decision-making problem of innovating firms. In particular they embed the agency model of Hart (1983) in an endogenous growth framework and show that competition has a positive effect on growth because, combined with the threat of bankruptcy, it can act as a discipline device, capable of fostering technology adoption and growth. However empirical evidence of these effects is mixed, as shown, for instance, by Grosfeld and Tressel (2001) and Nickell *et al.* (1997).

Another approach (see Aghion *et al.* 1997; Aghion *et al.* 2001) extends the basic Schumpeterian model by allowing incumbent firms to innovate. This is obtained by assuming a technological progress which is more "gradualist" ("step-by-step") than the standard models, where the leap-frogging of the previous incumbent is possible: innovation allows a firm to move one step ahead, with the lagging firm remaining active and eventually capable to catch up. In this models it is assumed that each intermediate good sector is characterized by a duopoly in which firms compete both in production and in R&D. Hence, since in this framework R&D is undertaken by the incumbents, the incentive to innovate depends not so much upon post-innovation rents per se, but more upon the difference between post-innovation and pre-innovation rents (the latter are equal to zero in the basic Schumpeterian model). In this case product market competition may act by reducing firms' pre-innovation rents more than it reduces their post-innovation rents. In other words, competition may increase the incremental profits from innovating and thereby encourage R&D investments aimed at escaping competition. This happens in those industries where both firms are technological par (leveled or neck-and-neck sectors), while in unleveled sectors the Schumpeterian effect of business stealing always prevails. The effect of an increase in product market competition on growth is ambiguous and depends on the size of innovation. If the latter is sufficiently large, the Schumpeterian effect always dominates; if it is sufficiently close to its lower bound, the escape competition effect prevails; finally for intermediate values the predicted relationship between competition and growth is an inverted-U-shape: the escape competition effect tends to dominate for low initial levels of competition, whereas the Schumpeterian effect tends to dominate at higher levels of competition. This prediction is in line with earlier findings of Scherer (1967), Levin *et al.* (1985) and others[1] and has also been tested by Aghion *et al.* (2005) using data from a panel of U. K. firms (the data run from 1973 to 1994). The same result is obtained by d'Aspremont *et al.* (2010) in a model where there are the possibility of multiple winners[2] of the patent race, asymmetric firms in the product market and imperfect patent protection.

Another attempt to show the existence of a nonmonotonic relationship between competition and growth can be found in Denicolò and Zanchettin[3] (2004). They build a Neoschumpeterian model in which they allow for several firms to be simultaneously active in each industry (because the innovation is non drastic) and identify circumstances (a large size of innovation or a high intensity of competition, or both) in which the productive efficiency effect (the reduction of total industry costs due to the fact that low-cost firms have a large portion of the market) dominates the business stealing effect. This and the presence of a front loading effect (in more competitive markets, a larger fraction of innovation rents accrues in the early stages of the innovative firm's life cycle) imply that the equilibrium rate of growth tends to increase with the intensity

[1]See Cohen and Levin (1989) for a brief survey of the empirical literature.

[2]The number of firms is endogenously determined and the set of successful ones is drawn by a Bernoullian random process.

[3]They measure competition as a switch from Cournot to Bertrand competition, so as a switch of the equilibrium price under the different regimes of competition.

of competition.

Recently also Acemoglu *et al.* (2012) provided a new explanation of the inverted-U shaped relationship between competition and growth based on the standardization process of the new technologies. Standardization is a costly process which is undertaken by newcomers: the lower is this cost, the higher is the intensity of competition. When standardization is very costly, growth is low because the new product does not enter the standardization process and so it is produced by employing skilled workers and this reduces the scale of production and the profitability. On the other hand, when standardization is cheap, the growth rate is still low because innovators enjoy ex post profits only for a short while.

In our paper we modify a standard quality ladder model and differently from Aghion *et al.* (2001) we assume that R&D is driven by outsider firms and the winners of the race sell licenses over their patents, instead of entering directly the intermediate good sector. As a reward they get the aggregate profit of the industry[4]. Moreover, we depart from Aghion *et al.* (2001) models because, instead of assuming a duopoly in which firms compete à la Bertrand, we suppose that in the intermediate good sector an unspecified number of firms compete à la Cournot (see Li and Yanagawa 2011 for an exhaustive brief survey on the existing literature on patent licensing, oligopoly and technological progress). These are quite natural assumptions. For example Mukherjee (2005) shows that when there is licensing after R&D and firms compete à la Cournot, the innovators will always undertake non-cooperative R&D. Moreover we assume that in the intermediate sector there are spillovers on costs in the form of strategic complementarities. Amir (1996, 2005a, 2005b) indagates exhaustively on games with strategic complementarities, in particular on the case of Cournot oligopoly: he brings Cournot equilibrium and the theory of super modular games together in order to enrich the results on Cournot equilibrium. The latter constitutes our key assumption. In fact our goal is to prove that there exists an interval of values of the spillover parameter such that the relationship between competition and growth is an inverted-U-shape, giving thus another theoretical foundation to the empirical evidence. In such case, when competition is low the spillover effect dominates the Schumpeterian business stealing effect and an increase in product market competition fosters growth. This is justified by the fact that incumbents firms may benefit from more competition as it increases the positive externality by a reduction of costs. When, instead, competition is high, the

[4]The analysis would be unchanged if we assume that the innovator gets a fraction of the profit.

business stealing effect prevails over the spillover effect. We use the number of firms in each sector as a measure of competition, thus an increase in competition is expressed by an increase in the number of competitors. We think that this is the most natural measure of competition in a Cournotian framework[5]. It is customary in macroeconomic literature to study the effect of competition by comparing economies with the same market structure, but different degrees of substitutability between differentiated goods (see Grossman and Helpman 1991; Aghion and Howitt 1992; Aghion *et al.* 2001; Barro and Sala-i-Martin 2004). In these types of models the inverse of the degree of substitutability coincides with the mark up. In our setup the mark up depends on both the degree of substitutability and the number of firms in each industry. Hence if the number of competitors increases then the mark up decreases, so that firms' market power reduces[6]. However we show in section 5 that our result is robust if we use the degree of substitutability as measure of competition. Moreover, the same applies when we endogenize the number of firms in the intermediate sector. In the industrial organization literature Belleflamme and Vergari (2011) study the relationship between different measures of competition (number of firms, degree of product differentiation, Cournot vs. Bertrand) and the incentive to innovate and find a non-monotonic relationship. In particular, under Cournot competition the incentive either decreases with the number of firms or is un inverted-U shape.

We think that this novel theoretical mechanism can actually provide an alternative, innovative and realistic explanation of the inverted-U-shaped relationship between competition and growth[7].

Our explanation hinges upon the presence of spillover effects in the intermediate sector, and there is a wide empirical literature which offers a substantial support to the idea that economies of scale are an important phenomenon both at aggregate and at sectorial level. For example, Basu and Fernald (1997), Sbordone (1997), Jimenez and Marchetti (2002) show that, for the U. S. economy, the overall level of returns to scale (in a Cobb- Douglas production function) should be placed in the interval $[1; 1.2]$, so

[5]See for example Motta (2004).

[6]The use of the degree of substitutability may also have undesirable effect: as stressed by Koeniger and Licandro (2006), a change in the elasticity of substitution modifies a fundamental parameter, which in turn may lead to different equilibrium allocations that cannot be straightforwardly compared across economies. In particular an increase in the degree of substitutability has only a reallocation effect which moves resources to the most efficient sector, without modifying relative prices. Hence this may lead to an overestimation of the impact of competition on the economy's growth rate.

[7]The presence of spillovers in manufacturing industries is well known and it is also proved empirically (see, e.g., the literature we report below).

that external increasing returns to scale should affect the economy's dynamics in the long run as well as in the short run. Increasing returns and economies of scale can give rise to (favorable) spillover effects in the firms cost functions. Moreover also the literature on knowledge spillover is abundant. For example there are many works about knowledge spillovers both at regional and international level[8]. Keller (2002) analyzes whether the scope of technological knowledge spillovers is global or local (the dataset encompasses the world's innovative activity between 1970 and 1995). He finds that the diffusion of technology is geographically localized, in accordance with the conclusions of Adams *et al.* (1993), Jaffe and Trajtenberg (1999) and Eaton and Kortum (1996; 1999). However this literature does not distinguish between inter and intra-sectoral spillovers. Our model supposes the existence of intra-sectoral spillovers and this assumption is also supported by empirical findings offered by the literature. Rouvinen (2002) analyzes Finland manufacturing firms over the period 1985-1997 and finds evidence about the existence of intra and inter-sectoral spillovers by estimating the variable cost function. On the other hand, Malerba *et al.* (2004), by means of a panel data analysis of six OECD countries in the 1981-1995 time interval, show that the effect of intra-sectoral knowledge spillovers is 70% higher than the effect of national inter-sectoral spillovers. Brandt (2007) estimates the cost function using data on manufacturing industries of six OECD countries over the period 1980-1998. His main findings are that knowledge spillovers explain some of the productivity growth observed and are identified as an external source of economies of scale. Moreover, international intra-industry spillovers are the most important source of externalities in the investigated industries: they turn out to be more significant than R&D spillovers. Finally, Badinger and Egger (2008), by considering 13 OECD countries and 15 manufacturing industries in the year 1995, find that knowledge spillovers occur both horizontally and vertically, whereas other types of productivity spillovers are primarily of the intra-industry type.

Also the empirical urban economic literature supports the presence of spillovers: it shows the importance for productivity and growth of localization economies[9] (economies of scale arising from spatial concentration economies) and urbanization economies[10] (economies of scale arising from city size itself). Rosenthal and Strange (2001), for example, test the microfoundation of agglomerations economies for U.S. four-digit SIC

[8] See, among the others, Coe and Helpman (1995), Van Stel and Nieuwenhuijsen (2004).

[9] See, e.g., Moomaw (1981), Sveikauskas (1975), Nakamura (1985), Henderson (1986) and Ciccone and Hall (1996).

[10] See, e.g., Glaeser *et al.* (1992) and Henderson *et al.* (1995).

codes manufacturing industries in the fourth quarter of 2000 at different levels of geographic aggregation and find that there is evidence of the importance of all sources of localization economies (the Marshall's three theories of industry agglomeration); in particular knowledge spillovers are relevant at the zipcode level, input sharing at state level and labor market pooling is important at all levels[11].

The paper is structured as follows. Section 2 describes the overall framework of the model and contains an interpretation of the consequences of the introduction of Cournot oligopoly (with spillover effects) in the intermediate sector. In Section 3 the steady-state expressions for the growth rate, the interest rate and the probability of innovation are derived. Section 4 discusses our main result: there exists an interval of values of the spillover parameter such that the relationship between competition and growth is an inverted-U-shape. Section 5 presents a numerical analysis for the UK economy, which is based on the calibration of the degree of substitutability between intermediate goods, the spillover intensity and the size of the leading-edge innovation. Moreover we show that the relationship between competition and growth is bell-shaped both if we consider the degree of substitutability between intermediates as a measure of competition and if we endogenize the number of firms. Finally, Section 6 concludes.

2 The model

2.1 The agents

Our starting point is the standard version of the Schumpeterian growth scheme as exposed in Barro, Sala-i-Martin (2004), Ch. 7 (a quality ladder model). In this scheme there are four types of agents in the economy. Producers of final good that use labor and intermediate goods input to produce output which is sold at a unit price and it is used for consumption, for the production of the intermediate goods and, finally, it is invested in R&D. The final good sector is perfectly competitive. R&D firms devote resources to discover a new quality of the existing intermediate good: once this one has been invented, the winner of the race obtains a perpetual patent. We modify this framework by considering the case in which R&D is undertaken by outsider firms. Moreover, the winning one can sell a given number of licenses for each sector to allow other firms to produce the quality-improved good. Thus the last one is an oligopolistic market and

[11]Also Ellison *et al.* (2010) assess the importance of all the Marshallian theories of industry agglomeration in U.S. three-digit SIC codes manufacturing industries from 1972 to 1997.

we assume that firms compete in quantity (Cournot competition). In particular we suppose that there exist m intermediate sectors (with m large) and in each sector there are n firms producing the same good; finally there are households who consume the final good and their saving finances R&D. The behavior of these agents will be detailed in the following sections.

2.2 Final good sector

The production function of the representative final good firm i is given by

$$Y_i = L_i^{1-\alpha} \sum_{h=1}^{m} \tilde{x}_{ih}^{\alpha}$$

where $0 < \alpha < 1$, Y_i is output, L_i is labor input, $\tilde{x}_{ih} = \sum_{j=1}^{n} \tilde{x}_{ihj} = \sum_{k=0}^{k_h} q^k x_{ihk}, q > 1$ represents the quality-adjusted amount employed of the hth type of intermediate good, h refers to the generic intermediate sector $h = 1,..,m$. The potential grades of each intermediate good are arrayed along a quality ladder with rungs spaced proportionately at interval $q > 1$. Fixing at 1 the beginning quality, the subsequent rungs are at the levels q, q^2 and so on. Thus, if k_h improvements in quality have occurred in sector h, the available grades in the sector are $1, q, q^2, ..., q^{k_h}$. Increases in k_h are possible thanks to the successful application of the research effort.

Hence the production function becomes:

$$Y_i = L_i^{1-\alpha} \sum_{h=1}^{m} \left(\sum_{k=0}^{k_h} q^k x_{ihk} \right)^{\alpha}$$

Assuming that $\forall h$ only the best quality is produced[12], the production function becomes:

$$Y_i = L_i^{1-\alpha} \sum_{h=1}^{m} q^{\alpha k_h} x_{ihk_h}^{\alpha}$$

Each firm seeks to maximize profit[13]:

$$\underset{\{L_i, x_{ihk_h}\}}{Max}\ \pi_i = Y_i - wL_i - \sum_{h=1}^{m} p_{hk_h} x_{ihk_h} = L_i^{1-\alpha} \sum_{h=1}^{m} q^{\alpha k_h} x_{ihk_h}^{\alpha} - wL_i - \sum_{h=1}^{m} p_{hk_h} x_{ihk_h}$$

[12]This will be proved in the following.

[13]We set the price of the final good equal to one.

The first order conditions are:

$$L_i = \left(\frac{1-\alpha}{w}\right)^{\frac{1}{\alpha}} \left(\sum_{h=1}^{m} q^{\alpha k_h} x_{ihk_h}^{\alpha}\right)^{\frac{1}{\alpha}}$$

$$p_{hk_h} = \alpha q^{\alpha k_h} x_{ihk_h}^{\alpha-1} L_i^{1-\alpha}$$

From the latter we get the demand function from firm i to sector h:

$$x_{ihk_h} = L_i q^{\frac{\alpha k_h}{1-\alpha}} \left(\frac{\alpha}{p_{hk_h}}\right)^{\frac{1}{1-\alpha}}$$

To find the total demand in sector h we have to aggregate for all i:

$$x_{hk_h} = \sum_i x_{ihk_h} = L q^{\frac{\alpha k_h}{1-\alpha}} \left(\frac{\alpha}{p_{hk_h}}\right)^{\frac{1}{1-\alpha}} \tag{2.1}$$

where $L = \sum_i L_i$ represents the aggregate labor force, assumed to be constant. The demand function for good produced in sector h is a decreasing function of the price.

To solve the Cournot problem in the intermediate goods sector, we need the aggregate inverse demand function:

$$p_{hk_h} = \alpha q^{\alpha k_h} x_{hk_h}^{\alpha-1} L^{1-\alpha} \tag{2.2}$$

2.3 Intermediate good sector

We assume that the winner of the R&D race does not produce directly the invention but sells the right to produce the new good to a given number of firms in each sector.

We suppose that there are m sectors and n firms in each one competing à la Cournot. We assume that in each sector h there are positive externalities or spillovers which are modeled as strategic complementarities: $\partial^2 \pi_{hj} / \left(\partial x_{hjk_h} \partial x_{hlk_h}\right) > 0, \forall i, l$. This means that the marginal profit of firm j increases as another competitor, say l, rises its produced quantity. This implies that firm j will find rising its quantity convenient.

A profit function satisfying this property is:

$\pi_{hj} = p_{hk_h} x_{hjk_h} - c\left(x_{hjk_h} / \left(\sum_{l \neq j} x_{hlk_h}\right)^{\gamma}\right), \gamma > 0$ where γ represents the spillover coefficient and $c(.)$ is a cost function. This assumption means that when a firm $l \neq j$ increases its production of the intermediate good, the production cost of firm j reduces.

This implies that the marginal revenue of j increases, so that the firm find it convenient to increase production.

We now specify the cost function. In the benchmark model the marginal cost of intermediate firms is one unit of final good[14]. In the present case, strategic complementarity implies that the marginal cost equals $1/\left(\sum_{l\neq j} x_{hlk_h}\right)^{\gamma}$. Defining $\sum_{l\neq j} x_{hlk_h} = x_{-j}$, the profit function of j is:

$$\pi_{hj} = p_{hk_h} x_{hjk_h} - \frac{x_{hjk_h}}{(x_{-j})^{\gamma}} \tag{2.3}$$

The cost function deserves some explanation, because, due to the hypothsis on the technology, the only way to include spillovers in this industry is to modify the marginal cost. In the benchmark model shown in Barro and Sala-i-Martin (2004), Ch. 7, each producer uses the same technology: one unit of the final good is needed to produce one unit of intermediate good, so that the marginal cost is equal to one. In order to include the spillover effect we need to divide the cost function by the quantity produced by competitors. Actually there is no other way to represent a spillover effect in this simple technological framewok, but to directly create a link with the quantities produced by rival firms. Nevertheless, several reasons may justify such cost-reduction effect as, for example, technological and intellectual spillovers between companies which are related to exchanges of information, skilled labor, etc. (an example of spillovers on the cost function which is similar to the one adopted here can be found in d'Aspremont and Jaquemin 1988. Moreover, in an industry populated by many firms producing an homogeneous good it is easy to find the presence of common infrastructural services which can certainly reduce the production cost of each single producer (this is an example of Marshallian externality). In our model the spillover effect is represented by the other firms' choice variables, and we excluded the quantity produced by the representative firm. This assumption is commonly used in the empirical literature on intra-industry spillovers on the cost function (see for example Bernstein and Nadiri 1989, Suzuki 1993, Rouvinen 2002). Nevertheless, empirical works estimating this type of cost function are rare, and this is due to the lack of high quality data and to the difficulty of estimating this specific functional form. In general, the empirical literature on spillovers can be splitted in two subgroups: the first one is the primal approach or technology flow, and the second one is the cost function or dual approach, which is intimately connected to advances in flexible functional forms. Actually, the estimations

[14]The final good is taken as the numeraire.

focus on these generalised functional forms, without any particular theory behind.

In a Cournot oligopoly, each firm chooses the quantity to be produced in order to maximize (2.3), where p_{hk_h} characterizes the inverse demand by the final good sector.

The resulting optimal price and quantity (we provide the derivations in the Appendix) are given by[15]

$$p^*_{hk_h} = \left\{ (n+\alpha-1)\frac{1}{n}\left[\left(\frac{n-1}{n}\right) Lq^{\frac{\alpha k_h}{1-\alpha}}\alpha^{\frac{1}{1-\alpha}}\right]^{\gamma}\right\}^{\frac{1-\alpha}{\alpha+\gamma-1}} \tag{2.4}$$

and

$$x^*(h) = x^*_{hjk_h} = \frac{1}{n}L^{\frac{\alpha-1}{\alpha+\gamma-1}} q^{-\frac{\alpha}{(\alpha+\gamma-1)}k_h}\alpha^{-\frac{1}{(\alpha+\gamma-1)}}\left\{(n+\alpha-1)\frac{1}{n}\left(\frac{n-1}{n}\right)^{\gamma}\right\}^{-\frac{1}{\alpha+\gamma-1}} \tag{2.5}$$

2.3.1 Comparative statics

Now we pass to examine the influence of the spillover parameter over the optimal quantity and price. The results are contained in the following Proposition.

Proposition 1. *If* $\left(L^{1-\alpha}q^{\alpha k_h}\alpha\,(n+\alpha-1)/n\right) > \left((n-1)/n\right)^{\alpha-1}$, *then the equilibrium quantity* (2.5) *is an increasing function of the spillover coefficient* γ *and the optimal price, which is given by* (2.4)*, is a decreasing function of* γ.

Proof. Consider the expression of the optimal quantity (2.5) and derive it with respect to γ so to obtain:

$$\begin{aligned}\frac{\partial x^*(h)}{\partial\gamma} \quad = \quad & \frac{1}{n}\left[L^{1-\alpha}q^{\alpha k_h}\frac{n+\alpha-1}{n}\right]^{\frac{1}{1-\alpha-\gamma}}\left(\frac{n-1}{n}\right)^{\frac{\gamma}{1-\alpha-\gamma}} \\ & \left[\frac{1}{(1-\alpha-\gamma)^2}Log\left(L^{1-\alpha}q^{\alpha k_h}\alpha\frac{n+\alpha-1}{n}\right)+\frac{1-\alpha}{(1-\alpha-\gamma)^2}Log\left(\frac{n-1}{n}\right)\right]\end{aligned}$$

The second term in the square bracket is negative. Hence if $\left(L^{1-\alpha}q^{\alpha k_h}\alpha\,(n+\alpha-1)/n\right) > \left((n-1)/n\right)^{\alpha-1}$, then $\partial x^*(h)/\partial\gamma$ is positive.

We also know that the price is decreasing in x_{hk_h}. If each oligopolist is rising its own output, then also the total quantity produced in sector h will increase, determining a fall in the price. □

[15] The second order conditions for a maximum are satisfied. In fact

$$SOC = \alpha(\alpha-1)\,q^{\alpha k_h}x^{*(\alpha-2)}_{hjk_h}L^{1-\alpha}\left[(\alpha-2)+2n\right]<0$$

The economic intuition is the following: if the spillover coefficient rises, this causes a reduction of costs for each firm, so that the output that equals marginal revenue to marginal cost must increase.

2.3.2 The MARK UP

Given that the optimal quantity and price are influenced by the spillover coefficient, we may expect that the mark up is also affected by γ. In this section we show that this does not happen.

We adopt the following definition of the mark up:

$$MU = \frac{P - MC}{MC} = \frac{P}{MC} - 1$$

where

$$MC = \frac{1}{(n-1)^{\gamma} x(h)^{*\gamma}}$$

is the marginal cost.

By using the expressions of the optimal quantity and price, we can rewrite MU in this way:

$$MU = \frac{1-\alpha}{n+\alpha-1}$$

Hence the mark up does not depend on the spillover parameter. In particular, it is equal to the mark up that we would obtain if strategic complementarities were absent[16].

Hence, the effects of γ on price and marginal cost must have the same magnitude, and this is due to the symmetry among the oligopolists. At a first glance, it may seem that the introduction of the spillover parameter in our model is irrelevant, but this is not the case: γ has nonetheless a sizable effect on both equilibrium price and quantity, as shown in the previous section. The fact is simply that, on one hand, γ has a negative impact on the equilibrium price and this implies a reduction of the mark up. But on the other hand, an increase in the spillover parameter reduces marginal costs MC, and this would imply an increase of the mark-up. The two effects are exactly balanced.

Taking the limit for n which tends to infinity, we find the usual property of the mark up:

[16]This can be proved by redoing the previous calculations with $\gamma = 0$.

$$\lim_{n\to+\infty}\frac{1-\alpha}{n+\alpha-1}=0$$

Finally, the mark up depends negatively on α, that is the degree of substitutability between the differentiated products, as in the standard quality-ladder model it is: $\partial MU/\partial\alpha=-n/\left(n+\alpha-1\right)^2<0$.

2.3.3 The optimal profit

Given the optimal quantity and price, we are able to compute the maximum profit for firm j in industry h:

$$\pi_{hj}^{*OLIG}=\bar{\pi}q^{\frac{\alpha(1-\gamma)}{1-\alpha-\gamma}k_h} \tag{2.6}$$

where

$\bar{\pi}=\left(\alpha L^{1-\alpha}\right)^{\frac{1-\gamma}{1-\alpha-\gamma}}\left[\left(n+\alpha-1\right)\left(1/n\right)\left(\left(n-1\right)/n\right)^{\gamma}\right]^{-\frac{\alpha}{\alpha+\gamma-1}}\left(1-\alpha\right)/n^2$. The optimal profit is positive for all $0<\alpha<1, n\geq 2, \gamma>0$. Moreover, we can note that $\lim_{n\to\infty}\pi_{hj}^{*OLIG}=0$.

2.3.4 The engine of growth

If we substitute (2.5) into the aggregate production function we obtain:

$$Y = L^{\frac{(1-\alpha)(\gamma-1)}{\gamma+\alpha-1}}\alpha^{-\frac{\alpha}{\alpha+\gamma-1}}\left(\frac{1}{n}\right)^{\alpha}\left[\left(n+\alpha-1\right)\frac{1}{n}\left(\frac{n-1}{n}\right)^{\gamma}\right]^{-\frac{\alpha}{\alpha+\gamma-1}}\sum_{h=1}^{m}q^{\frac{\alpha(\gamma-1)}{\alpha+\gamma-1}k_h}$$

We define $Q\left(\gamma\right)\equiv\sum_{h=1}^{m}q^{\left(\alpha(\gamma-1)k_h/(\alpha+\gamma-1)\right)}$ as the *Adjusted* aggregate quality index[17], so that the last equation can be rewritten in this way:

$$Y=L^{\frac{(1-\alpha)(\gamma-1)}{\gamma+\alpha-1}}\alpha^{-\frac{\alpha}{\alpha+\gamma-1}}\left(\frac{1}{n}\right)^{\alpha}\left[\left(n+\alpha-1\right)\frac{1}{n}\left(\frac{n-1}{n}\right)^{\gamma}\right]^{-\frac{\alpha}{\alpha+\gamma-1}}Q\left(\gamma\right)$$

The key element in fostering the growth of aggregate output turns out to be the dynamics of the quality-ladder positions, k_h, in the various sectors. The impact of $Q\left(\gamma\right)$ is amplified by the spillover effect represented by γ, as the exponent of q in $Q\left(\gamma\right)$ is an

[17]The term "adjusted" is justified by the fact that with respect to the basic model in this case the spillover parameter appears.

increasing function of γ. We should expect this effect because of the influence of the externality on the optimal quantities of intermediate goods.

2.4 The R&D sector

2.4.1 Modeling destruction

In the previous sections we assumed that only the best quality k_h of the intermediate good h would be produced and used in each intermediate industry: this implies that the innovation process is drastic.

We now pass to investigate under which condition a drastic innovation occurs.

The different intermediate goods are perfect substitutes but are weighted by their respective grades, and each unit of the leading-edge good is equivalent to q units of the good of the previous quality. Thus, if the state of the art is sold at a price given by(2.4), the next best quality will be sold, at most, at the price p_{hk_h}/q. As a consequence, the following relationship holds: $p_{hk_h-1} \leq p_{hk_h}/q = MC = 1/x_{-j}^{\gamma}$, and when a drastic innovation occurs, it must be: $p_{hk_h}/q < 1/x_{-j}^{\gamma}$.

By substituting (2.4) and (2.5), we obtain:

$$\frac{n}{n+\alpha-1} < q$$

Note that it is: $q^{-1} < 1$, while it is $n/\left(n+\alpha-1\right) > 1$; furthermore, the term $n/\left(n+\alpha-1\right)$ is decreasing in n. Thus for a high enough n, the inequality $n/\left(n+\alpha-1\right) < q$ is satisfied and the right-hand-side is also decreasing in α.

We finally note that the fact of having drastic innovation or not does not depend on the degree of spillover[18].

2.4.2 Modeling creation

We consider an endogenous Poisson process. This means that the time which should be waited for innovation to occur is a random variable which is distributed as an exponential. The parameter of this distribution constitutes the arrival rate of the Poisson process. We assume that it depends positively on the R&D aggregate expenditure in sector h, z_{hk_h}, and negatively on k_h for a given z_{hk_h}: the negative impact of z_{hk_h} is due to the increasing difficulty in innovation after the initial and easier stages. The flow probability to move from k_h to k_{h+1} is equal to:

[18] The same justification we gave for the mark up independence from the spillover parameter applies.

$$p\left(k_h\right) = z_{hk_h}\varphi\left(k_h\right)$$

Hence probability p is an endogenous variable, because the level of R&D effort is chosen by the R&D firms.

2.4.3 Determination of R&D effort (steady-state analysis)

We assume that R&D is undertaken by outsiders, and in order to obtain the research arbitrage condition (and to determine p), the cost of R&D activity must be equated to respective benefits. A successful innovation grants an infinitely lived patent, hence the benefits of innovation are given by the flow of profits starting from the moment of innovation and discounted by the cumulative interest factor and the probability to be replaced by another innovation. By equating costs and benefits we obtain:

$$z_{hk_h} = p\left(k_h\right) nE\left(\pi_{hjk_h+1}\right)$$

Actually, once an outsider R&D firm succeeds in innovating, it obtains a perpetual patent, whose expected value is equal to: $E\left(\pi_{hjk_h+1}\right)$, which is subsequently sold as license to the n firms in the intermediate sector h. Thus, as a reward, the innovator obtains the entire aggregate profit of the industry h**:**

$$z_{hk_h} = z_{hk_h}\varphi\left(k_h\right)\int_t^{+\infty} n\pi^{*OLIG}_{hjk_h+1}e^{-r(\tau-t)}e^{-p(k_h+1)(\tau-t)}d\tau$$

If we assume that the economy grows along a steady state path, then the interest rate is constant and the former equation can be recast in this way:

$$1 = \varphi\left(k_h\right) n\frac{\bar{\pi}q^{\frac{\alpha(1-\gamma)(k_h+1)}{1-\alpha-\gamma}}}{r+p\left(k_h+1\right)}$$

$$r+p\left(k_h+1\right) = \varphi\left(k_h\right) n\bar{\pi}q^{\frac{\alpha(1-\gamma)(k_h+1)}{1-\alpha-\gamma}}$$

We need now to specify the functional form of $\varphi\left(k_h\right)$. We assume constant returns to scale in the relationship between the rate of return of R&D $\left(r+p\left(k_h+1\right)\right)$ and the demand-driven effect (coming from final good producers) which is represented by the term $q^{(\alpha(1-\gamma)(k_h+1)/(1-\alpha-\gamma))}$ (recall that aggregate output is proportional to the latter factor). Thus we adopt the following specification: $\varphi\left(k_h\right) = q^{-(\alpha(1-\gamma)(k_h+1)/(1-\alpha-\gamma))}/\eta$, where η is a parameter representing the cost of doing research. In other words, a successful innovation becomes more difficult the greater the output that would be produced at the newly attained ladder position $k_h + 1$[19].

Given this assumption, the research arbitrage condition turns out to be equal to:

$$r + p\left(k_h + 1\right) = n\frac{\bar{\pi}}{\eta}$$

or also

$$p = n\frac{\bar{\pi}}{\eta} - r \tag{2.7}$$

So that if r is constant over time, then p also is constant.

3 The growth process

We assume Ramsey consumers, so that the growth rate of consumption is equal to

$$g = \frac{\dot{c}}{c} = \frac{1}{\sigma}\left(r - \rho\right) \tag{3.1}$$

where $\frac{1}{\sigma}$ is the intertemporal elasticity of substitution and $\rho > 0$ is the discount rate[20].

Given that this is a lab-equipment model, the market clearing condition, $Y = C + X + Z$, which states that output is consumed and used in the production of intermediate goods (X) and in R&D (Z), implies that all the terms are proportional to $Q\left(\gamma\right)$ and so $g_C = g_Y = g_X = g_Z = g_Q = g$.

To compute the growth rate of $Q\left(\gamma\right)$, we first consider what happens in each sector h, then, by applying the law of large number, we describe the economy in the aggregate.

The proportional increase in quality in each sector is: $q^{(\alpha(1-\gamma)(k_h+1)/(1-\alpha-\gamma))} - q^{(\alpha(1-\gamma)k_h/(1-\alpha-\gamma))}/q^{(\alpha(1-\gamma)k_h/(1-\alpha-\gamma))} = q^{\alpha(1-\gamma)/(1-\alpha-\gamma)} - 1$. In aggregate terms, the expected proportional increase of quality is:

$$g = \frac{\dot{Q}\left(\gamma\right)}{Q\left(\gamma\right)} = p\left(q^{\frac{\alpha(1-\gamma)}{1-\alpha-\gamma}} - 1\right) \tag{3.2}$$

We assume that $q^{\alpha(1-\gamma)/(1-\alpha-\gamma)} - 1 > 0$, so it must be that $\left(1-\gamma\right)/\left(1-\alpha-\gamma\right) > 0$. We thus obtain a system of three equations, (2.7), (3.1) and (3.2) in three unknowns,

[19] This is a commonly used function (see Barro and Sala-i-Martin 2004, p. 327).

[20] We assume that the population growth rate is equal to zero.

r, g and p.

Solving the system. By solving the system composed by (2.7), (3.1) and (3.2) , we obtain the steady-state expressions for g, r and p as a function of the model's parameters:

$$g=\frac{n\frac{\bar{\pi}}{\eta}-\rho}{1+\sigma\left(q^{\frac{\alpha(1-\gamma)}{1-\alpha-\gamma}}-1\right)}\left(q^{\frac{\alpha(1-\gamma)}{1-\alpha-\gamma}}-1\right) \tag{3.3}$$

where $\bar{\pi}=\left(\alpha L^{1-\alpha}\right)^{(1-\gamma)/(1-\alpha-\gamma)}\left[(n+\alpha-1)\,(1/n)\,((n-1)\,/n)^{\gamma}\right]^{-\alpha/(\alpha+\gamma-1)}(1-\alpha)\,/n^{2}$,

$$r=\frac{n\frac{\bar{\pi}}{\eta}\sigma\left(q^{\frac{\alpha(1-\gamma)}{1-\alpha-\gamma}}-1\right)+\rho}{1+\sigma\left(q^{\frac{\alpha(1-\gamma)}{1-\alpha-\gamma}}-1\right)}$$

$$p=\frac{n\frac{\bar{\pi}}{\eta}-\rho}{1+\sigma\left(q^{\frac{\alpha(1-\gamma)}{1-\alpha-\gamma}}-1\right)}$$

The growth rate, as given by (3.3), depends negatively on the households' preference parameters, ρ and σ, and on the R&D cost. On the other hand, it is an increasing function of $\bar{\pi}$ and q.

Before discussing the conditions required for having a positive growth rate g, recall that it must be: $q^{\alpha(1-\gamma)/(1-\alpha-\gamma)}-1>0$ and, as consequence, $\alpha\,(1-\gamma)\,/\,(1-\alpha-\gamma)>0$; this inequality provides a first constraint on the parameters' values and determines also the presence of the usual scale effect.

4 The relationship between competition and growth

In order to analyze the relationship between competition and growth, we must however check that the balanced growth path is feasible: this in turns implies that some sufficient conditions on the model's parameters have to be satisfied for having a positive g.

We first derive the steady-state growth rate with respect to the number of firms n in the intermediate good sector, which is the chosen measure of competition[21]. Our results are summarized in the following proposition:

[21] In Appendix B we show that the results are preserved also when the number of firms is considered as a discrete variable.

Proposition 2. *Suppose that the number of firms is a continuous variable. If* $\gamma > 1, n \geq 2$*, the balanced growth rate* g *in equation* (3.3) *is a decreasing function of the level of competition in each intermediate sector, as measured by* n*. If* $\gamma < 1-\alpha$*, the steady state growth rate is an inverted-U-shape function of* n *for* $\gamma \in \left((1-\alpha)/(1+\alpha)^2, 1-\alpha\right)$*, while for* $\gamma \in \left(0, (1-\alpha)/(1+\alpha)^2\right)$*, the balanced growth rate is still a decreasing function of* n*.*

Proof. In order to analyze the sign of the derivative of the growth rate with respect to the degree of competition, it is sufficient to compute the derivative of $n\bar{\pi}(n)$ with respect to n, since this is the unique term of g which depends on n. Hence:

$$sign\left(\frac{\partial g}{\partial n}\right) = sign\left(\frac{\partial\left(n\bar{\pi}(n)\right)}{\partial n}\right)$$

Differentiating
$n\bar{\pi}(n) = \bar{\pi} = \left(\alpha L^{1-\alpha}\right)^{(1-\gamma)/(1-\alpha-\gamma)}\left[(n+\alpha-1)(1/n)((n-1)/n)^{\gamma}\right]^{-\alpha/(\alpha+\gamma-1)}(1-\alpha)/n$
with respect to n yields

$$\begin{aligned}\frac{\partial(n\bar{\pi})}{\partial n} &= \frac{1-\alpha}{n^2}\left(\alpha L^{1-\alpha}\right)^{\frac{1-\gamma}{1-\alpha-\gamma}}\left[\frac{n+\alpha-1}{n}\left(\frac{n-1}{n}\right)^{\gamma}\right]^{-\frac{\alpha}{\alpha+\gamma-1}} \\ &\cdot\left\{-\frac{\alpha}{\alpha+\gamma-1}\left[\frac{1-\alpha}{n+\alpha-1}+\gamma\frac{1}{n-1}\right]-1\right\} \end{aligned} \tag{4.1}$$

The sign of this derivative depends on the sign of the curly bracket, which in turn depends on the term $-\alpha/(\alpha+\gamma-1)$. We must distinguish two cases:

- If $\gamma > 1-\alpha$, i.e.: the spillovers are sufficiently high, then $\{-\alpha/(\alpha+\gamma-1)[(1-\alpha)/(n+\alpha-1)+\gamma/(n-1)]-1\} < 0$, thus $\partial g/\partial n < 0$. However, we should exclude the values of the spillover parameter in the interval: $\gamma \in (1-\alpha, 1)$, otherwise the BGP will not be feasible.

- If $\gamma < 1-\alpha$, that is the degree of spillover is relatively low, then the sign of $\{-\alpha/(\alpha+\gamma-1)[(1-\alpha)/(n+\alpha-1)+\gamma/(n-1)]-1\}$ is ambiguous. In order to make it clearer, we analyze the sign of this derivative in correspondence of the lower bound of the number of firms: $n=2$. In particular

$$\begin{aligned}\frac{\partial(n\bar{\pi})}{\partial n}|_{n=2} &= \frac{1-\alpha}{4}\left(\alpha L^{1-\alpha}\right)^{\frac{1-\gamma}{1-\alpha-\gamma}}\left[\frac{1+\alpha}{2}\left(\frac{1}{2}\right)^{\gamma}\right]^{-\frac{\alpha}{\alpha+\gamma-1}} \\ &\cdot\left\{-\frac{\alpha}{\alpha+\gamma-1}\left[\frac{1-\alpha}{1+\alpha}+\gamma\right]-1\right\} > 0\end{aligned}$$

if and only if $\{-\alpha/(\alpha+\gamma-1)[(1-\alpha)/(1+\alpha)+\gamma]-1\} > 0$. This occurs when $\gamma > (1-\alpha)/(1+\alpha)^2 \equiv \tilde{\gamma}$. Note that $\tilde{\gamma} \in (0, 1-\alpha)$. Moreover $\lim_{n\to\infty}(n\bar{\pi}) = 0$. Thus if $\gamma \in (\tilde{\gamma}, 1-\alpha)$ the relationship between competition and growth is nonmonotonic: it is increasing for small values of n and decreasing for large values of n. When instead it is $\gamma \in (0, \tilde{\gamma})$, the function $n\bar{\pi}(n)$ is decreasing in a neighborhood of $n = 2$ and for $n \to \infty$.

It remains to understand the behavior of this function in the interval $n \in (2, +\infty)$. To this aim we propose the following argument. The derivative (4.1) is equal to zero if and only if $-\alpha/(\alpha+\gamma-1)[(1-\alpha)/(n+\alpha-1)+\gamma/(n-1)]-1 = 0$ which is a second order equation in n:

$$(1-\alpha-\gamma)n^2 - 2(\alpha\gamma-\alpha-\gamma+1)n + (1-\alpha)(1-\gamma-\alpha\gamma) = 0$$

This equation admits two real roots. In fact, by computing the discriminant we found that it is equal to $\alpha^2(1-\alpha)\gamma > 0, \forall 0 < \alpha < 1, \gamma > 0$. We should now check whether these roots are greater or smaller than 2.

In order to do this, we study the product and the sum of the solutions, which are given by $n_1n_2 = (1-\alpha)(1-\gamma+\alpha\gamma)/(1-\alpha-\gamma)$ and $n_1+n_2 = 2(\alpha\gamma-\alpha-\gamma+1)/(1-\alpha-\gamma)$. They are both positive as we are in the region where $\gamma < 1-\alpha$, so that the roots are greater than zero. Moreover, it can be shown that in our case

$$n_1 + n_2 > 2 \text{ and } 1 < n_1n_2 < 2 \tag{4.2}$$

We now have to distinguish between two cases:

1- If $\gamma \in (\tilde{\gamma}, 1-\alpha)$, we know that the function $n\bar{\pi}(n)$ is increasing in a neighborhood of $n = 2$ and $\lim_{n\to+\infty} n\bar{\pi}(n) = 0$. Thus we can find a unique global maximum in the interval $(2, +\infty)$, while the other stationary point must be smaller than 2, in order to satisfy (4.2). We can conclude that the shape of the balanced growth rate as a function of competition is an inverted U in the relevant interval.

2- If $\gamma \in (0, \tilde{\gamma})$, we can immediately note that the function can not attain a minimum and then a maximum in the interval $(2, +\infty)$, otherwise conditions (4.2) would not be satisfied; in particular it would be that $n_1n_2 > 2$ and if conditions (4.2) must be satisfied, the case in which it is $n_1 > 2$ and $n_2 > 2$ must be excluded. Hence, we are left with only two possibilities: i) one of the two stationary points is greater than 2 and in this case it must be a flex with an horizontal tangent; ii) both n_1 and n_2 are smaller than 2.

In both cases the function turns out to be monotonically decreasing in the interval $(2, +\infty)$, and under ii) it is strictly decreasing.

These considerations conclude the proof. $\square$

The economic intuition of this result is the following. There are two ways of fostering spillovers: an increase in γ, which represents the intensity of the external economies of scale and an increase of the number of firms in each industry, which determines an increase in the aggregate quantity produced by the whole industry and so a reduction of each firm's marginal cost. Here, for a fixed γ, we study the effect of a change in the number of firms. Suppose that the spillovers are high. Then existing firms in the intermediate good sector would not be favored by an increase in strategic complementarities due to the entrance of new firms, as the incumbents are already big: the unique consequence would be a reduction of profits.

On the other hand, if spillovers are relatively low, then it can be possible that for a low number of firms the spillover effect dominates the business stealing effect because the few existing firms would benefit from more competition as it increases the strategic complementarities. But when n rises beyond a certain threshold, the business stealing effect prevails again, inducing a decline of the steady state growth rate. In this case, the relationship between competition and growth is an inverted-U-shape. Hence for low values of the spillovers parameters, when the number of firms is small enough, the spillover effect is greater than the business stealing effect. This interpretation can be supported by the following considerations. Consider the model without strategic complementarities, i. e. $\gamma = 0$. In this case $\bar{\pi}$ becomes

$$\bar{\pi}_{\gamma=0} = \left(\alpha L^{1-\alpha}\right)^{\frac{1}{1-\alpha}} \left[(n+\alpha-1)\,\frac{1}{n}\right]^{-\frac{\alpha}{\alpha-1}} \left[\frac{1-\alpha}{n^2}\right]$$

As a consequence

$$\frac{\partial \bar{\pi}_{\gamma=0}}{\partial n} = \left(\alpha L^{1-\alpha}\right)^{\frac{1}{1-\alpha}} \left[(n+\alpha-1)\,\frac{1}{n}\right]^{-\frac{\alpha}{\alpha-1}} \left[\frac{1-\alpha}{n^4}\right] \left[\frac{\alpha n}{n+\alpha-1} - 1\right] < 0$$
$$\forall 0 < \alpha < 1, n \geq 2$$

and

$$\frac{\partial (n\bar{\pi})_{\gamma=0}}{\partial n} = \left(\alpha L^{1-\alpha}\right)^{\frac{1}{1-\alpha}} \left[(n+\alpha-1)\,\frac{1}{n}\right]^{-\frac{\alpha}{\alpha-1}} \left[\frac{1-\alpha}{n^2}\right] \left[\frac{\alpha}{n+\alpha-1} - 1\right] < 0$$
$$\forall 0 < \alpha < 1, n \geq 2$$

Thus if there were no spillovers in the intermediate good sector, then the relationship between competition and growth would be negative.

We now introduce the remaining conditions which guarantee the positivity of the balanced growth rate.

Proposition 3. *The balanced growth rate g, which is given by expression* (3.3)*, is positive if* $(1-\alpha)/2\left(\alpha L^{1-\alpha}\right)^{(1-\gamma)/(1-\alpha-\gamma)}\left[(1+\alpha)/2\,(1/2)^{\gamma}\right]^{-\alpha/(\alpha+\gamma-1)} > \eta\rho$*. If this condition is satisfied, then it is possible to identify a closed, compact set of admissible values for the firms' number in the intermediate good sector, which are also sustainable in the long run.*

Proof. We previously assumed that $q^{\alpha(1-\gamma)/(1-\alpha-\gamma)} - 1 > 0$. So, in order to have a positive long run growth rate, it must be: $\bar{\pi} > \eta\rho/n$, i.e.: $n\bar{\pi} > \eta\rho$. Define $n\bar{\pi} = h(n)$ and $\eta\rho \equiv i(n) = i$, which is a constant function with respect to n. Function $h(n)$ is continuous in n for $n > 1$[22] and it is monotonically decreasing if $\gamma > 1$. When instead it is $\gamma < 1-\alpha$, $h(n)$ is increasing (w.r.t. n) and then decreasing if γ is in the interval $\gamma \in \left((1-\alpha)/(1+\alpha)^2, 1-\alpha\right)$; finally, for $\gamma \in \left(0, (1-\alpha)/(1+\alpha)^2\right)$, $h(n)$ is decreasing , as shown in the previous proposition.

We now provide a sufficient condition on the parameters ensuring that i lies below $h(n)$ for $n = 2$: $h(2) > i$. This imply that, by continuity, the two functions must cross at least once, let us say in $\bar{\bar{n}}$.

The sufficient condition for having a positive BGP growth rate and a compact, closed set of firms that can survive in the long run $\left([2, \bar{\bar{n}}]\right)$ is:

$$\begin{aligned} h(2) &= \left(\alpha L^{1-\alpha}\right)^{\frac{1-\gamma}{1-\alpha-\gamma}}\left[\tfrac{1+\alpha}{2}\left(\tfrac{1}{2}\right)^{\gamma}\right]^{-\frac{\alpha}{\alpha+\gamma-1}}\left[1-\tfrac{1+\alpha}{2}\right] \\ &= \left(\alpha L^{1-\alpha}\right)^{\frac{1-\gamma}{1-\alpha-\gamma}}\left[\tfrac{1+\alpha}{2}\left(\tfrac{1}{2}\right)^{\gamma}\right]^{-\frac{\alpha}{\alpha+\gamma-1}}\tfrac{1-\alpha}{2} > \eta\rho \equiv i \end{aligned}$$

that is

$$\frac{1-\alpha}{2}\left(\alpha L^{1-\alpha}\right)^{\frac{1-\gamma}{1-\alpha-\gamma}}\left[\frac{1+\alpha}{2}\left(\frac{1}{2}\right)^{\gamma}\right]^{-\frac{\alpha}{\alpha+\gamma-1}} > \eta\rho$$

This concludes the proof. □

This proposition identifies an upper bound for the sustainable number of firms in the long run, which includes the scale effect of endogenous growth models: the larger is L, the greater is the growth rate and the upper bound of the sustainable interval of n. Furthermore, the lower are η or ρ, the larger is the admissible number of firms, and these two parameters also have a negative impact on the growth rate.

[22]We remark that we are interested in $n \geq 2$.

5 Calibration

5.1 The spillover parameter

We now adopt our framework for calibrating the values of the spillover parameter and the parameter representing the size of the leading-edge innovation for the UK economy. We have chosen this methodology taking into account the fact that it is difficult to get high quality data on firms' costs because the same firms has an incentive to keep this information private. Anyways our numerical analysis is sufficient to prove the existence and the importance in terms of relationship between competition and growth of intra-industry spillovers.

We use UK data, so to be consistent with Aghion *et al.* (2005) seminal paper[23]. We also need to calibrate the income share of intermediate goods α, because estimations of a production function with only labor and intermediates are not present in the literature. To this aim, we use the equation of the mark up, $MU = (1-\alpha)/(n+\alpha-1)$, along the lines of Aghion *et al.* (2005); they use the price-cost margin[24] as a measure of product market competition, which is an approximation of the Lerner index. As the quantification of marginal costs is notoriously difficult, Aghion *et al.* (2005) approximate the price-cost margin with the ratio between operating profits (net of the financial costs) and sales. To compute this quantity, they use a panel of 311 firms of seventeen two-digit SIC codes industries over the period 1973-1994. The average Lerner index is 4%, which yields a mark up of 4.2%. Finally, by using the average number of firms of these sectors, we obtain $\alpha = 0.263$.

We calibrate the steady state interest rate r through equation (3.1). To this end we set $g = 2.18\%$ (source: World Bank, 1973-1994), $\rho = -\log\beta = -\log 0.99 = 0.01$ (source: DSGE literature; see for example King and Rebelo 2000), $\sigma = 1$[25], so to obtain $r = 3.18\%$.

[23] Most of the empirical works on the relationship between competition and growth are based on UK data because the United Kingdom experienced a large number of policy changes that led to exogenous variation in the nature and magnitude of competition.

[24] Price-cost margin is defined as the difference between price and marginal cost divided by price.

[25] If we consider a greater value for the inverse of the intertemporal elasticity of substitution, this does not change the conclusion on the spillover parameter. The same applies if we consider a different discount rate, for example $\rho = 0.03$. Thus our analysis is robust to changes in parameter values.

Finally, we use the remaining steady state equations:

$$\begin{cases} g = p\left(q^{\frac{\alpha(1-\gamma)}{1-\alpha-\gamma}} - 1\right) \\ p = n\frac{\bar{\pi}}{\eta} - r \end{cases}$$

calibrate γ and q.

In order to measure η we choose the average Industry R&D expenditures (by performer) over GDP, which is equal to 0.0143 (source: National Science Foundation, 1975-1992). We then set $L = 1$ and $p = 0.04$, consistently with the estimation performed by Caballero and Jaffe (2002).

Hence the resulting calibrated parameters are equal to:

1. $\gamma = 0.5782 \in (0.462, 0.737) \equiv \left(\frac{1-\alpha}{(1+\alpha)^2}, 1-\alpha\right)$;
2. $q = 1.8652$.

These results support our theoretical model: since $\gamma \in \left((1-\alpha)/(1+\alpha)^2, 1-\alpha\right)$, the relationship between competition and growth is inverted-U shape for the UK economy.

5.2 The degree of substitability as a measure of competition

Earlier studies consider the degree of substitability α between the intermediates in order to analyze the relationship between competition and growth (see Grossman and Helpman 1991; Aghion and Howitt 1992; Aghion et al. 2001; Barro and Sala-i-Martin 2004). Given the complexity of expression (3.3), we can not provide an analytical result, but a numerical example can show that the analytical result of proposition 2 is robust. In order to plot equation (3.3) as a function of α we use the calibration of section 5.1:

n	$311/17$
ρ	0.01
σ	1
L	1
η	0.0143
γ	0.5782
q	1.8652

Table 1: Baseline calibration

Figure 5.1 shows that the relationship between competition and growth, as measured by an increase of the degree of substitutability between the intermediates , and growth is bell-shaped:

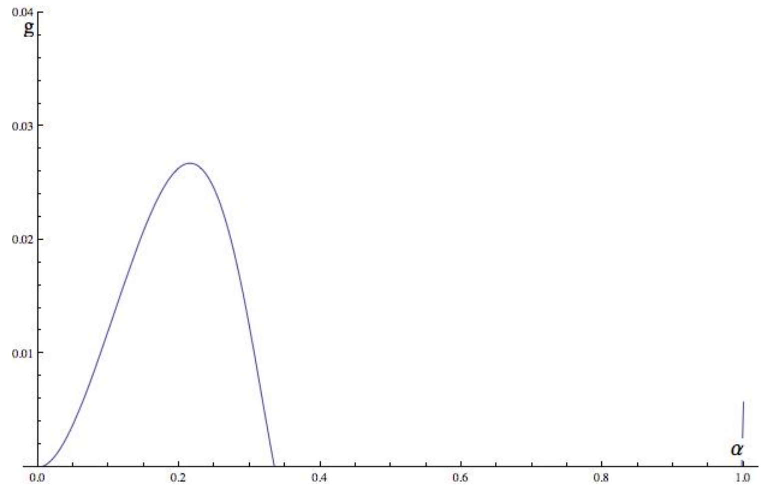

Figure 5.1: The degree of substitutability as a measure of competition

5.3 Endogenizing the number of firms

Up to now we assumed that the number of firms in the intermediate sector is exogenous. It may be natural to think that an inventor decides how many firms to license her innovation to in order to maximize her profit. Hence the R&D firm solves the following problem:

$$\max_n \quad z_{hk_h}\varphi\left(k_h\right)\int_t^{+\infty} n\pi^{*OLIG}_{hjk_h+1}e^{-r(\tau-t)}e^{-p(k_h+1)(\tau-t)}d\tau$$

which is equivalent to

$$\max_n \quad n\pi^{*OLIG}_{hjk_h+1}$$

where $\pi^{*OLIG}_{hjk_h+1}$ is given by (2.6). Since the only element related to n is $n\bar{\pi}\left(n\right)$, the first order condition is given by[26]

$$\frac{\partial\left(n\bar{\pi}\right)}{\partial n}=\frac{1-\alpha}{n^2}\left(\alpha L^{1-\alpha}\right)^{\frac{1-\gamma}{1-\alpha-\gamma}}\left[\frac{n+\alpha-1}{n}\left(\frac{n-1}{n}\right)^{\gamma}\right]^{-\frac{\alpha}{\alpha+\gamma-1}}\left\{-\frac{\alpha}{\alpha+\gamma-1}\left[\frac{1-\alpha}{n+\alpha-1}+\gamma\frac{1}{n-1}\right]-1\right\}$$

We know from proposition 2 that

- if $\gamma>1$ or $\gamma\in\left(0,\left(1-\alpha\right)/\left(1+\alpha\right)^2\right)$, function $n\bar{\pi}\left(n\right)$ is decreasing for $n\in[2,+\infty)$. The optimal number of firms is therefore 2;

[26]See the proof of proposition 2.

- if $\gamma \in \left((1-\alpha)/(1+\alpha)^2, 1-\alpha\right)$, function $n\bar{\pi}(n)$ is bell-shaped for $n \in [2, +\infty)$. In this case $\arg\max\{n\bar{\pi}(n)\}$ solves

$$(1-\alpha-\gamma)n^2 - 2(\alpha\gamma - \alpha - \gamma + 1)n + (1-\alpha)(1-\gamma-\alpha\gamma) = 0$$

and the optimal number of firms turns out to be[27]:

$$n^* = \frac{(\alpha\gamma - \alpha - \gamma + 1) + \sqrt{(\alpha\gamma - \alpha - \gamma + 1)^2 - (1-\alpha)(1-\alpha-\gamma)(1-\gamma-\alpha\gamma)}}{(1-\alpha-\gamma)}$$

When n is endogenous comparative statics on competition should therefore be done by varying the degree of substitutability between the intermediate goods. By using the baseline calibration of table 5.1 we find that in both cases the relationship between competition and growth is an inverted-U shape (Figure 5.2):

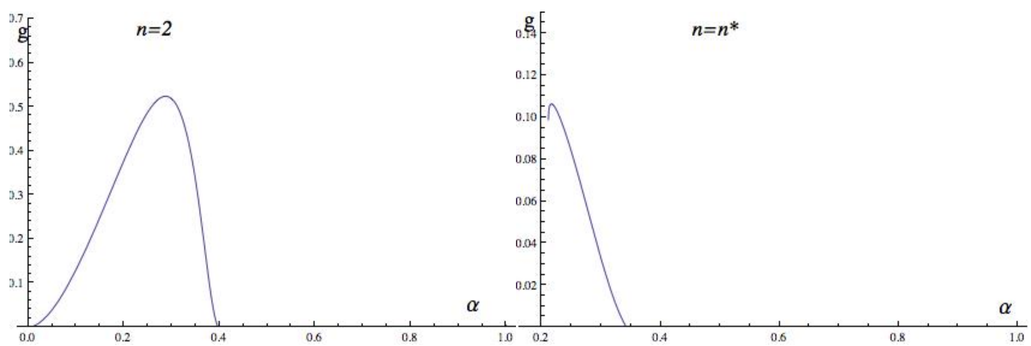

Figure 5.2: n endogenous

6 Conclusions

Empirical evidence suggests the presence of an inverted U-shaped relationship between competition and growth. But early models of endogenous growth show that a stronger competition erodes the innovator's prospective monopoly rent and reduces the incentive to innovate. Only recently theory was able to explain the nonmonotonicity of the above relationship. Our model can be viewed as another attempt to justify it from a theoretical point of view. We found a set of circumstances under which the behavior of the growth rate as a function of the number of firms in each industry switches form increasing to decreasing. The growth rate increases with the number of firms for small

[27]See the proof of proposition 2 for futher details.

degrees of competition, as the spillover effect dominates the business-stealing effect; when competition becomes tougher, and the Schumpeterian effect of a reduction of profits prevails, the growth rate decreases with the number of firms.

By applying our model to the UK data, for the 1973-1994 period, we found that the calibrated value of the spillover parameter lies in the region where the the relationship between competition and growth is non-monotonic.

We then recast the analysis by using the degree of substitutability between the intermediates as measure of competition and we proved the robustness of our result. Finally we endogenized the number of firms and found that the relationship between competition and growth is still an inverted-U shape.

These considerations may provide a rationale for antitrust policies aimed at fostering competition in innovative sectors: in industries where the strategic complementarities are not too strong and not too weak, policy makers should enhance competition in order to reach a higher growth rate.

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A Derivations of the optimal price and the optimal quantity

Each representative firm in sector h solves the following problem

$$\max_{x_{hjk_h}} \pi_{hj} = p_{hk_h} x_{hjk_h} - \frac{x_{hjk_h}}{(x_{-j})^{\gamma}}$$

The first order conditions are:

$$\begin{gathered}\frac{\partial \pi_{hj}}{\partial x_{hjk_h}} = 0 \Rightarrow \\ p'_{hk_h} x_{hjk_h} + p_{hk_h} = \frac{1}{(x_{-j})^{\gamma}}\end{gathered}$$

Summing all the first order conditions for all firms in an industry (the sum over j allows us to use the aggregate demand function $x_{hk_h} = \sum_i x_{ihk_h}$), we obtain:

$$\begin{aligned} p'_{hk_h} \sum_{j=1}^{n} x_{hjk_h} + n p_{hk_h} &= \frac{n}{(x_{-j})^{\gamma}} \\ p'_{hk_h} x_{hk_h} + n p_{hk_h} &= \frac{n}{(x_{-j})^{\gamma}} \\ p_{hk_h} &= \frac{1}{(x_{-j})^{\gamma}} - \frac{1}{n} p'_{hk_h} x_{hk_h} \end{aligned}$$

Equation (2.2) can now be derived with respect to x_{hk_h}:

$$\begin{aligned} p'_{hk_h} &= \alpha(\alpha-1) q^{\alpha k_h} x_{hk_h}^{\alpha-2} L^{1-\alpha} \\ p'_{hk_h} x_{hk_h} &= \alpha(\alpha-1) q^{\alpha k_h} x_{hk_h}^{\alpha-1} L^{1-\alpha} \end{aligned}$$

By using (2.1), the last equation turns out to be equal to:

$$p'_{hk_h} x_{hk_h} = \alpha(\alpha-1) q^{\alpha k_h} \left(L q^{\frac{\alpha k_h}{1-\alpha}} \left(\frac{\alpha}{p_{hk_h}} \right)^{\frac{1}{1-\alpha}} \right)^{\alpha-1} L^{1-\alpha} = (\alpha-1) p_{hk_h} \tag{A.1}$$

Now consider the term $1/(x_{-j})^{\gamma}$. By definition it is: $x_{hk_h} = \sum_{j=1}^{n} x_{hjk_h}$, while, by the assumption of symmetry, it is $x_{hk_h} = \sum_{j=1}^{n} x_{hjk_h} = n x_{hjk_h} \Rightarrow x_{hjk_h} = \frac{1}{n} x_{hk_h}$. So $x_{-j} = \sum_{l \neq j} x_{hlk_h} = (n-1)/n x_{hk_h}$. These facts, together with expression (2.1). allow us to write:

$$\begin{aligned}\frac{1}{(x_{-j}\ )^{\gamma}} &= \left(\frac{n}{n-1}\right)^{\gamma}\frac{1}{(x_{hk_h})^{\gamma}} = \left(\frac{n}{n-1}\right)^{\gamma}\left(Lq^{\frac{\alpha k_h}{1-\alpha}}\left(\frac{\alpha}{p_{hk_h}}\right)^{\frac{1}{1-\alpha}}\right)^{-\gamma} \\ &= \left(\frac{n}{n-1}\right)^{\gamma}L^{-\gamma}q^{-\frac{\gamma\alpha k_h}{1-\alpha}}\alpha^{-\frac{\gamma}{1-\alpha}}p_{hk_h}^{\frac{\gamma}{1-\alpha}}\end{aligned} \tag{A.2}$$

By plugging (A.1) and (A.2) into the sum of the first order conditions of industry h, we obtain:

$$p_{hk_h} = \left(\frac{n}{n-1}\right)^{\gamma}L^{-\gamma}q^{-\frac{\gamma\alpha k_h}{1-\alpha}}\alpha^{-\frac{\gamma}{1-\alpha}}p_{hk_h}^{\frac{\gamma}{1-\alpha}} - \frac{1}{n}(\alpha-1)\,p_{hk_h}$$

which can be divided by p_{hk_h}

$$\begin{aligned}1 &= \left(\frac{n}{n-1}\right)^{\gamma}L^{-\gamma}q^{-\frac{\gamma\alpha k_h}{1-\alpha}}\alpha^{-\frac{\gamma}{1-\alpha}}p_{hk_h}^{\frac{\gamma+\alpha-1}{1-\alpha}} - \frac{1}{n}(\alpha-1) \\ n &= n\left(\frac{n}{n-1}\right)^{\gamma}L^{-\gamma}q^{-\frac{\gamma\alpha k_h}{1-\alpha}}\alpha^{-\frac{\gamma}{1-\alpha}}p_{hk_h}^{\frac{\gamma+\alpha-1}{1-\alpha}} - (\alpha-1) \\ n+\alpha-1 &= n\left(\frac{n}{n-1}\right)^{\gamma}L^{-\gamma}q^{-\frac{\gamma\alpha k_h}{1-\alpha}}\alpha^{-\frac{\gamma}{1-\alpha}}p_{hk_h}^{\frac{\gamma+\alpha-1}{1-\alpha}} \\ p_{hk_h}^{\frac{\gamma+\alpha-1}{1-\alpha}} &= (n+\alpha-1)\frac{1}{n}\left(\frac{n-1}{n}\right)^{\gamma}L^{\gamma}q^{\frac{\gamma\alpha k_h}{1-\alpha}}\alpha^{\frac{\gamma}{1-\alpha}}\end{aligned}$$

Thus the optimal price is

$$p^{*}_{hk_h} = \left\{(n+\alpha-1)\frac{1}{n}\left[\left(\frac{n-1}{n}\right)Lq^{\frac{\alpha k_h}{1-\alpha}}\alpha^{\frac{1}{1-\alpha}}\right]^{\gamma}\right\}^{\frac{1-\alpha}{\alpha+\gamma-1}}$$

This expression allows us to compute the optimal quantity produced by each firm in h. By the assumption of symmetry, it is

$$x_{hjk_h} = \frac{1}{n}x_{hk_h}$$

$$
\begin{aligned}
x_{hjk_h} &= \frac{1}{n} L q^{\frac{\alpha k_h}{1-\alpha}} \left(\frac{\alpha}{p_{hk_h}}\right)^{\frac{1}{1-\alpha}} \\
&= \frac{1}{n} L q^{\frac{\alpha k_h}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} \left\{ (n+\alpha-1) \frac{1}{n} \left[\left(\frac{n-1}{n}\right) L q^{\frac{\alpha k_h}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} \right]^{\gamma} \right\}^{\frac{1-\alpha}{\alpha+\gamma-1}\left(-\frac{1}{1-\alpha}\right)}
\end{aligned}
$$

Thus the equilibrium quantity produced by the sector h oligopolists is equal to:

$$
x^*(h) = x^*_{hjk_h} = \frac{1}{n} L^{\frac{\alpha-1}{\alpha+\gamma-1}} q^{-\frac{\alpha}{(\alpha+\gamma-1)} k_h} \alpha^{-\frac{1}{(\alpha+\gamma-1)}} \left\{ (n+\alpha-1) \frac{1}{n} \left(\frac{n-1}{n}\right)^{\gamma} \right\}^{-\frac{1}{\alpha+\gamma-1}}
$$

B The discrete case

In Section 4 we considered the number of firms as a continuous variable. Actually $n \in \mathbb{N}$, thus both the domain and the codomain of the growth rate, $g(n)$, are numerable. In the following proposition we show that our main result is preserved in this case.

Proposition 4. *If the number of firms in each intermediate sector is such that* $n \in \mathbb{N}, n \geq 2$*, then when* $\gamma > 1$*, the steady state growth rate* g *in equation* (3.3) *is a decreasing function of* n*, while when* $\gamma < 1-\alpha$*, the relationship between competition and growth is an inverted-U-shape function if*
$\gamma \in \left((1-\alpha\left[\log(2+\alpha) - \log(1+\alpha)\right] / (\log 3 - \log 2)) / \left(1+\alpha\left(\log \frac{4}{3} / \log \frac{3}{2}\right)\right), 1-\alpha \right)$,
while it is monotonically decreasing if
$\gamma \in \left(0, (1-\alpha\left[\log(2+\alpha) - \log(1+\alpha)\right] / (\log 3 - \log 2)) / \left(1+\alpha\left(\log \frac{4}{3} / \log \frac{3}{2}\right)\right)\right)$.

Moreover, there exists the following link between the sufficient conditions in the continuous and discrete case that guarantee the non monotonicity of the above relationship:

$$
\begin{gathered}
\left((1-\alpha\left[\log(2+\alpha) - \log(1+\alpha)\right] / (\log 3 - \log 2)) / \left(1+\alpha\left(\log \frac{4}{3} / \log \frac{3}{2}\right)\right), 1-\alpha \right) \subset \\
\left((1-\alpha) / (1+\alpha)^2, 1-\alpha \right)
\end{gathered}.
$$

Proof. We have proved that, if n is a continuous variable, when $\gamma > 1$, the growth rate is monotonically decreasing in the number of firms. For this values of γ, the monotonicity is thus preserved when $n \in \mathbb{N}$.

We now focus on the case in which is $\gamma < 1-\alpha$. Consider again the function $n\bar{\pi}(n) = \bar{\pi} = \left(\alpha L^{1-\alpha}\right)^{(1-\gamma)/(1-\alpha-\gamma)} \left[(n+\alpha-1)(1/n)((n-1)/n)^{\gamma}\right]^{-\alpha/(\alpha+\gamma-1)} (1-\alpha)/n$ and compute the first difference

$$
\begin{gathered}
(n+1)\,\overline{\pi}\,(n+1) - n\overline{\pi}\,(n) = \\
(1-\alpha)\left(\alpha L^{1-\alpha}\right)^{\frac{1-\gamma}{1-\alpha-\gamma}}\left\{\left[(n+\alpha)\,\tfrac{1}{n+1}\left(\tfrac{(n+1)-1}{n+1}\right)^{\gamma}\right]^{-\frac{\alpha}{\alpha+\gamma-1}}\tfrac{1}{n+1}\right. \\
\left. -\left[(n+\alpha-1)\,\tfrac{1}{n}\left(\tfrac{n-1}{n}\right)^{\gamma}\right]^{-\frac{\alpha}{\alpha+\gamma-1}}\tfrac{1}{n}\right\}
\end{gathered}
$$

We now compute it for $n = 2$ and determine the (sufficient) condition on γ as a function of α for which it is: $3\pi\,(3) - 2\overline{\pi}\,(2) > 0$:

$$
\frac{2}{3} > \left[\frac{2+\alpha}{3}\left(\frac{2}{1+\alpha}\right)\left(\frac{4}{3}\right)^{\gamma}\right]^{\frac{\alpha}{\alpha+\gamma-1}}
$$

By solving for γ we obtain:

$$
\gamma > \frac{1-\alpha\frac{[\log(2+\alpha)-\log(1+\alpha)]}{\log 3-\log 2}}{1+\alpha\left(\frac{\log\frac{4}{3}}{\log\frac{3}{2}}\right)} \equiv \hat{\gamma}
$$

The value $\hat{\gamma}$ is lower than $1-\alpha$, as it can be shown by inspecting the graph of the function $f\,(\alpha) = \hat{\gamma} + \alpha$ when $0 < \alpha < 1$ (Figure B.1):

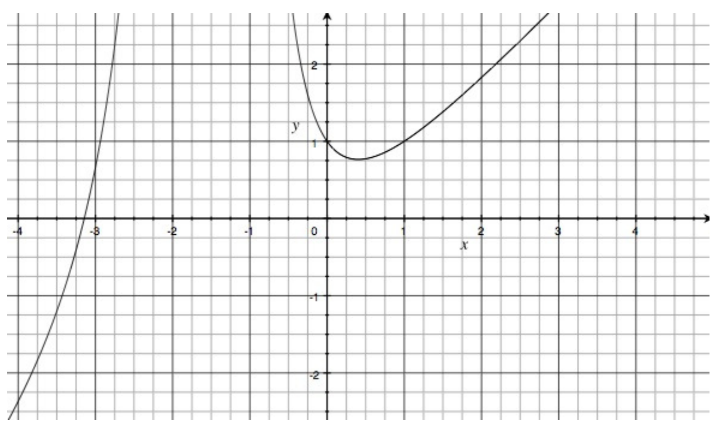

Figure B.1: $f\,(\alpha)$

On the other hand, the lower bound of $\hat{\gamma}$ is $(1-\alpha)\,/\,(1+\alpha)^2$, as it can be checked from the graph of $g\,(\alpha) = \hat{\gamma} - (1-\alpha)\,/\,(1+\alpha)^2$ (for $0 < \alpha < 1$) (Figure B.2):

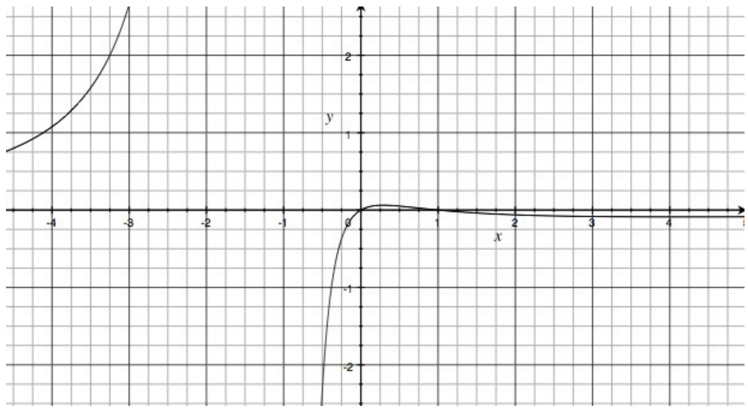

Figure B.2: $g(\alpha)$

Thus we showed that
$\left((1-\alpha\left[\log(2+\alpha)-\log(1+\alpha)\right]/(\log 3-\log 2))/\left(1+\alpha\left(\log\frac{4}{3}/\log\frac{3}{2}\right)\right),1-\alpha\right)\subset\left(\frac{(1-\alpha)}{(1+\alpha)^2},1-\alpha\right)$.

Furthermore, we know that $n\bar{\pi}(n)$ in the continuous case becomes decreasing after a certain n. This behavior is preserved in the discrete case, confirming the inverted-U-shape feature of the relationship between the growth rate and the number of firms.

On the other hand, if $\gamma\in(0,\hat{\gamma})$, the quantity $3\pi(3)-2\bar{\pi}(2)$ is negative. In the continuous case we showed that for very small values for γ the function $n\bar{\pi}(n)$ is decreasing for $n\in(2,+\infty)$. This implies that when γ lies in this interval, our function is decreasing in the discrete case too.

This concludes the proof. $\square$

We should remark that in the proof of Proposition 3 we made use of the fact that the growth rate is a continuous function of n. Actually both the functions $n\bar{\pi}\equiv h(n)$ and $\eta\rho\equiv i(n)\equiv i$ are discrete in n. However, for the growth rate to be positive, an inequality is needed, so that we can disregard the intersection between the two functions.